New constraints in dynamical torsion theory

M. O. Katanaev *

Steklov Mathematical Institute,
Gubkin St., 8, 117966, Moscow, Russia

16 September 1992

Abstract

The most general Lagrangian for dynamical torsion theory quadratic in curvature and torsion is considered. We impose two simple and physically reasonable constraints on the solutions of the equations of motion: (i) there must be solutions with zero curvature and nontrivial torsion and (ii) there must be solutions with zero torsion and non covariantly constant curvature. The constraints reduce the number of independent coupling constants from ten to five. The resulting theory contains Einstein's general relativity and Weitzenböck's absolute parallelism theory as the two sectors.

1 Introduction

Dynamical torsion theory (known also as a Poincaré gauge theory) is the simplest geometric generalization of general relativity in which torsion as well as metric is an independent dynamical variable [1]. The most general ten parameter invariant Lagrangian yielding second order equations of motion contains invariants quadratic in curvature and torsion. Large number of independent coupling constants raises a crucial question what choice of the coupling constants are most acceptable from physical point of view?

There have been different approaches to get restrictions on the coupling constants in dynamical torsion theory. Unfortunately, there are weak experimental bounds (see, for example, [2, 3]) and one has to take into account theoretical considerations. In [4–6] propagators for all modes entering the theory were considered and Lagrangians without ghosts and tachyons were found. Closely related criterion of positivity of masses and energy contributions to the canonical Hamiltonian in the linear approximation was considered in [7, 8]. Another constraints on the coupling

*E-mail: katanaev@mi.ras.su

constants can be found by the validity of Birkhoff theorem [9–11]. The restrictions on the coupling constants of the quadratic torsion terms can be obtained by the requirement of asymptotically Newtonian behavior of the gravitational field [12]. The initial value problem and the corresponding restrictions on the coupling constants following from the validity of Cauchy–Kowalevski theorem as well as the hyperbolicity conditions were found in [13, 14]. To obtain the above restrictions on the coupling constants one has to make cumbersome calculations. Therefore, simpler restrictions are plausible.

In the present paper we impose new simple but very restrictive constraints on the Lagrangian. Namely, we demand equations of motion to admit solutions (i) with zero curvature and nontrivial torsion (absolute parallelism) and (ii) with zero torsion and non covariantly constant curvature (Einsteinian limit). We prove that only five parameter Lagrangian satisfies these constraints. The resulting Lagrangian is the sum of the Hilbert–Einstein Lagrangian for tetrad field, the three invariants quadratic in curvature, and cosmological constant. The three quadratic in curvature terms entering the Lagrangian are those that vanish in the case of zero torsion and describe massless Lorentz connection. It is very interesting that the resulting Lagrangian, after further fixing one more constant and dropping the cosmological constant, coincides with the unique Lagrangian without ghosts and tachyons found by Kuhfuss and Nitsch [6].

These constraints are motivated by the following consideration. Equations of motion in a general dynamical torsion theory have a very particular form. If one sets torsion equal zero in the equations of motion then one obtains the Einstein equations for a metric and additional constraint on the curvature tensor that is absent in general relativity. The last follows from the equation for the Lorentz connection and states that the curvature must be covariantly constant. Thus if one wants the usual general relativity to be incorporated in dynamical torsion theory then one should try to find the set of coupling constants that admits solutions of zero torsion and non covariantly constant curvature.

For zero curvature the equation for the Lorentz connection in a general case yields zero torsion condition, the theory becoming trivial. Thus if one wants the Weitzenböck absolute parallelism theory [15, 16] to be also incorporated in dynamical torsion theory then one should find the coupling constants that admit solutions of zero curvature and nontrivial torsion. It is quite surprising that the set of coupling constants admitting general relativity and absolute parallelism theory as the two limiting cases of dynamical torsion theory exists.

The constraints used to reduce the number of coupling constants have also direct physical interpretation in solids with defects. It is known that curvature and torsion are correspondingly the surface densities of Frank and Burgers vectors for continuously distributed dislocations and disclinations [17–20]. Thus the two imposed constraints are nothing more than existence of the space-times with only dislocations or only disclinations. Since dislocations and disclinations in media are occurred independently, one may say that the constraints have experimental background.

The paper is organized as follows. In Section 2 the equations of motion are derived for a general type action. In Section 3 we solve the constraints and derive the five parameter Lagrangian. The Conclusion contains brief discussion of the resulting model.

2 General type action

Let $\mathbb{R}^4$ be a four-dimensional manifold (space-time) with coordinates x^μ, $\mu = 0,1,2,3$. We assume that the space-time is a Riemann–Cartan manifold equipped with metric $g_{\mu\nu}(x) = g_{\nu\mu}(x)$ and torsion $T_{\mu\nu}{}^\rho(x) = -T_{\nu\mu}{}^\rho(x)$ [21]. Equivalent realization of Riemann–Cartan geometry can be given in terms of the tetrad $e_\mu{}^a$, $a = 0,1,2,3$ and the Lorentz connection $\omega_\mu{}^{ab} = -\omega_\mu{}^{ba}$ [22]. Torsion and curvature have the following form in terms of these variables

$$\begin{aligned} T_{\mu\nu}{}^a &= \partial_\mu e_\nu{}^a - \omega_\mu{}^{ab} e_{\nu b} - (\mu \leftrightarrow \nu), \\ R_{\mu\nu}{}^{ab} &= \partial_\mu \omega_\nu{}^{ab} - \omega_\mu{}^{ac} \omega_{\nu c}{}^b - (\mu \leftrightarrow \nu). \end{aligned}$$

Here and in what follows transition between "curved" Greek and "flat" Latin indices is performed using the tetrad. Greek indices are raised and lowered by the metric $g_{\mu\nu}$, while Latin indices are raised and lowered by the Lorentz metric η_{ab} =diag$(+---)$.

The most general invariant action yielding second order equations of motion for tetrad and Lorentz connection contains ten arbitrary parameters

$$I = \int d^4x eL, \qquad e = \det e_\mu{}^a, \tag{1}$$

where

$$\begin{aligned} L \;=\;& \kappa R - \frac{1}{4} T_{abc} \left(\beta_1 T^{abc} + \beta_2 T^{cab} + \beta_3 \eta^{ac} T^b \right) \\ & - \frac{1}{4} R_{abcd} \left(\alpha_1 R^{abcd} + \alpha_2 R^{cdab} + \alpha_3 R^{acbd} + \alpha_4 \eta^{bd} R^{ac} + \alpha_5 \eta^{bd} R^{ca} \right) + \lambda. \end{aligned} \tag{2}$$

Ricci tensor, scalar curvature, and the trace of torsion are defined as follows $R_{ac} = R_{abc}{}^b$, $R = R_a{}^a$, $T_b = T_{ab}{}^a$. Lagrangian (2) contains all independent invariants constructed from T_{abc}, R_{abcd} and containing no more than two partial derivatives [23].

Let us note two identities which exclude two possible invariants from the Lagrangian. One has no need to add the Hilbert–Einstein Lagrangian $\widetilde{R}(e)$ constructed only from tetrad (scalar curvature of zero torsion) because of the identity

$$R + \frac{1}{4} T_{abc} T^{abc} - \frac{1}{2} T_{abc} T^{cab} - T_a T^a - \frac{2}{e} \partial_\mu (e T^\mu) = \widetilde{R}(e). \tag{3}$$

The second identity is the Gauss–Bonnet formula which excludes scalar curvature squared term

$$-R_{abcd} R^{cdab} + 4 R_{ab} R^{ba} - R^2 = \frac{1}{e} \partial_\mu (\dots). \tag{4}$$

In the next section we will also use the Bianchi identities[1]

$$
\begin{aligned}
\nabla_\mu R_{\nu\rho}{}^{ab} + \nabla_\nu R_{\rho\mu}{}^{ab} + \nabla_\rho R_{\mu\nu}{}^{ab} =& T_{\mu\nu}{}^{\sigma} R_{\rho\sigma}{}^{ab} + T_{\nu\rho}{}^{\sigma} R_{\mu\sigma}{}^{ab} + T_{\rho\mu}{}^{\sigma} R_{\nu\sigma}{}^{ab}, && (5)\\
\nabla_\mu T_{\nu\rho}{}^{a} + \nabla_\nu T_{\rho\mu}{}^{a} + \nabla_\rho T_{\mu\nu}{}^{a} =& T_{\mu\nu}{}^{\sigma} T_{\rho\sigma}{}^{a} + T_{\nu\rho}{}^{\sigma} T_{\mu\sigma}{}^{a} + T_{\rho\mu}{}^{\sigma} T_{\nu\sigma}{}^{a}\\
&+ R_{\mu\nu\rho}{}^{a} + R_{\nu\rho\mu}{}^{a} + R_{\rho\mu\nu}{}^{a}, && (6)
\end{aligned}
$$

where ∇ denotes the covariant derivative with Lorentz connection for Latin indices and the corresponding metrical connection $\Gamma_{\mu\nu}{}^{\rho}$ for Greek indices. The latter is defined by equation

$$
\nabla_\mu e_\nu{}^a = \partial_\mu e_\nu{}^a - \omega_\mu{}^a{}_b e_\nu{}^b - \Gamma_{\mu\nu}{}^{\rho} e_\rho{}^a = 0. \tag{7}
$$

Equations of motion for action (1) have the form

$$
\begin{aligned}
\frac{1}{e}\frac{\delta I}{\delta e_\mu{}^a} =& \kappa\left(R e^\mu{}_a - 2R_a{}^\mu\right) + \beta_1\left(\widetilde{\nabla}_\rho T^{\rho\mu}{}_a - \frac{1}{4}T_{bcd}T^{bcd}e^\mu{}_a + T^{\mu bc}T_{abc}\right)\\
&+\beta_2\left[-\frac{1}{2}\widetilde{\nabla}_\nu (T_a{}^{\mu\nu} - T_a{}^{\nu\mu}) - \frac{1}{4}T_{bcd}T^{dbc}e^\mu{}_a - \frac{1}{2}T^{b\mu c}T_{cab} + \frac{1}{2}T^{bc\mu}T_{cab}\right]\\
&+\beta_3\left[-\frac{1}{2}\widetilde{\nabla}_\nu (T^\nu e^\mu{}_a - T^\mu e^\nu{}_a) - \frac{1}{4}T_b T^b e^\mu{}_a + \frac{1}{2}T^\mu T_a + \frac{1}{2}T^b T_{ab}{}^\mu\right]\\
&+\alpha_1\left(-\frac{1}{4}R_{bcde}R^{bcde}e^\mu{}_a + R^{\mu bcd}R_{abcd}\right)\\
&+\alpha_2\left(-\frac{1}{4}R_{bcde}R^{debc}e^\mu{}_a + R^{cd\mu b}R_{abcd}\right)\\
&+\alpha_3\left(-\frac{1}{4}R_{bcde}R^{bdce}e^\mu{}_a + \frac{1}{2}R_{abcd}R^{\mu cbd} - \frac{1}{2}R_{abcd}R^{bc\mu d}\right)\\
&+\alpha_4\left(-\frac{1}{4}R_{bc}R^{bc}e^\mu{}_a + \frac{1}{2}R^{bc}R_{bac}{}^\mu + \frac{1}{2}R_{ab}R^{\mu b}\right)\\
&+\alpha_5\left(-\frac{1}{4}R_{bc}R^{cb}e^\mu{}_a + \frac{1}{2}R^{cb}R_{bac}{}^\mu + \frac{1}{2}R_{ab}R^{b\mu}\right) + \lambda e^\mu{}_a = 0,
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
\frac{1}{e}\frac{\delta I}{\delta \omega_\mu{}^{ab}} =& \kappa\left(T_a e^\mu{}_b + \frac{1}{2}T_{ab}{}^\mu\right) + \beta_1\frac{1}{2}T^\mu{}_{ba} + \beta_2\frac{1}{4}\left(T_a{}^\mu{}_b - T_{ab}{}^\mu\right) + \beta_3\frac{1}{4}T_b e^\mu{}_a\\
&+\alpha_1\frac{1}{2}\widetilde{\nabla}_\nu R^{\nu\mu}{}_{ab} + \alpha_2\frac{1}{2}\widetilde{\nabla}_\nu R_{ab}{}^{\nu\mu} + \alpha_3\frac{1}{4}\widetilde{\nabla}_\nu\left(R^\nu{}_a{}^\mu{}_b - R^\mu{}_a{}^\nu{}_b\right)\\
&+\alpha_4\frac{1}{4}\widetilde{\nabla}_\nu\left(R^\nu{}_a e^\mu{}_b - R^\mu{}_a e^\nu{}_b\right) + \alpha_5\frac{1}{4}\widetilde{\nabla}_\nu\left(R_a{}^\nu e^\mu{}_b - R_a{}^\mu e^\nu{}_b\right)\\
&-(a\leftrightarrow b) = 0.
\end{aligned} \tag{9}
$$

Here the tilde sign over the covariant derivative means that it acts with Christoffell's symbols (metrical connection of zero torsion) $\widetilde{\Gamma}_{\mu\nu}{}^{\rho}$ on Greek indices and with Lorentz

[1] In the journal version of the paper the covariant derivative in the Bianchi identities was understood with the Lorentz connection acting on Latin indices and the Christoffel symbol acting on Greek ones. In that case terms with torsion in the right hand sides are abcent.

connection $\omega_\mu{}^{ab}$ on Latin ones. For example,

$$\widetilde{\nabla}_\rho T^{\nu\mu}{}_a = \partial_\rho T^{\nu\mu}{}_a + \widetilde{\Gamma}_{\rho\sigma}{}^\nu T^{\sigma\mu}{}_a + \widetilde{\Gamma}_{\rho\sigma}{}^\mu T^{\nu\sigma}{}_a - \omega_{\rho a}{}^b T^{\nu\mu}{}_b.$$

The difference between Greek and Latin indices arises after integration by parts because of the identity $\partial_\mu e = e\widetilde{\Gamma}_{\nu\mu}{}^\nu$.

To analyse the equations of motion we decompose torsion and curvature into irreducible components. Torsion tensor has three irreducible components

$$T_{abc} = t_{abc} + \epsilon_{abcd}T^{*d} + \frac{1}{3}(\eta_{ac}T_b - \eta_{bc}T_a),$$

where we have extracted totally antisymmetric part by the use of totally antisymmetric tensor ϵ_{abcd}, $\epsilon_{0123} = 1$, and the trace

$$\begin{aligned} T^{*d} &= -\frac{1}{6}T_{abc}\epsilon^{abcd}, \qquad T_b = T_{ab}{}^a, \\ t_{abc} &= -t_{bca}, \qquad t_{ab}{}^a = 0, \qquad t_{abc} + t_{bca} + t_{cab} = 0. \end{aligned}$$

Curvature tensor has six irreducible components

$$\begin{aligned} R_{abcd} &= C^S_{abcd} + C^A_{abcd} + \frac{1}{2}\left(R^S_{ac}\eta_{bd} - R^S_{ad}\eta_{bc} - R^S_{bc}\eta_{ad} + R^S_{bd}\eta_{ac}\right) \\ &\quad + \frac{1}{2}\left(R^A_{ac}\eta_{bd} - R^A_{ad}\eta_{bc} - R^A_{bc}\eta_{ad} + R^A_{bd}\eta_{ac}\right) + \frac{1}{12}R\left(\eta_{ac}\eta_{bd} - \eta_{ad}\eta_{bc}\right) + \epsilon_{abcd}R^*, \end{aligned}$$

where we have extracted totally antisymmetric part R^* and all traces

$$\begin{aligned} R^* &= -\frac{1}{24}R_{abcd}\epsilon^{abcd}, \\ R^S_{ab} &= \frac{1}{2}\left(R_{ab} + R_{ba}\right) - \frac{1}{4}R\eta_{ab}, \qquad R^A_{ab} = \frac{1}{2}\left(R_{ab} - R_{ba}\right), \\ C_{abcd} &= -C_{bacd} = -C_{abdc}, \qquad C_{abc}{}^b = 0, \qquad C_{abcd} + C_{acdb} + C_{adbc} = 0, \\ C^S_{abcd} &= \frac{1}{2}\left(C_{abcd} + C_{cdab}\right), \qquad C^A_{abcd} = \frac{1}{2}\left(C_{abcd} - C_{cdab}\right). \end{aligned}$$

Irreducible component C^A_{abcd} can be parametrized by a symmetric traceless pseudotensor

$$C^A_{abcd} = -\frac{1}{8}(\epsilon_{abce}D_d{}^e - \epsilon_{abde}D_c{}^e - \epsilon_{cdae}D_b{}^e + \epsilon_{cdbe}D_a{}^e),$$

where

$$D_{de} = D_{ed} = C^A_{abcd}\epsilon^{abc}{}_e, \qquad D_d{}^d = 0.$$

The symmetry of D_{de} can be easily checked by multiplying its definition by totally antisymmetric tensor.

Note that for zero torsion three irreducible components of curvature equal zero

$$C^A_{abcd} = 0, \qquad R^A_{ab} = 0, \qquad R^* = 0,$$

and C^S_{abcd} is called the Weyl tensor. In this case the curvature tensor has additional symmetry

$$R_{abcd} = R_{cdab}. \tag{10}$$

3 Solution of the constraints

In the present section we impose two constraints on the space of solutions of equations of motion which allow one to determine five of the ten parameters entering the Lagrangian (2). The first constraint states that there must be solutions with zero curvature and nontrivial torsion. The following theorem yields the restrictions on coupling constants.

Theorem 1. *Equation of motion (9) has solution* $R_{abcd} = 0,\ t_{abc} \neq 0,\ T^{*a} \neq 0,\ T_a \neq 0$ *if and only if the coupling constants obey*

$$\beta_1 = -\kappa, \qquad \beta_2 = 2\kappa, \qquad \beta_3 = 4\kappa, \tag{11}$$

$\alpha_1, \ldots, \alpha_5$ *being arbitrary.*

Proof. Extracting the trace and totally antisymmetric part of equation (9) for $R_{abcd} = 0$ one obtains the equations

$$\begin{aligned} (8\kappa - 2\beta_1 + \beta_2 - 3\beta_3)T_a &= 0, \\ (\kappa - \beta_1 - \beta_2)T^{*a} &= 0, \\ (4\kappa + 2\beta_1 - \beta_2)t_{abc} &= 0. \end{aligned} \tag{12}$$

Because all irreducible components of torsion are not equal to zero one can easily check that Eqs.(11) are the unique solution of Eqs.(12). □

Under the restrictions on the coupling constants (11) the first four terms in the Lagrangian (2) yield the Hilbert–Einstein Lagrangian due to the identity (3). For this choice of the coupling constants and zero curvature, equation of motion for the Lorentz connection is identically satisfied while equation of motion for the tetrad (8) reduces to the Einstein equation

$$\widetilde{R}_{\mu\nu} - \frac{1}{2}\widetilde{R}g_{\mu\nu} - \frac{\lambda}{2\kappa}g_{\mu\nu} = 0. \tag{13}$$

Note that the tetrad field satisfying this equation defines nontrivial torsion on the manifold while the curvature remains zero. This model is known as Weitzenböck gravity theory with absolute parallelism (or teleparallelism theory) [15, 16].

The second constraint on the space of solutions is that there must be solutions with zero torsion and non covariantly constant curvature. Corresponding restrictions on the coupling constants are given by the following theorem.

Theorem 2. *Equation of motion (9) has solution* $T_{abc} = 0,\ \nabla_\mu R \neq 0,\ \nabla_\mu C_{abc}{}^\mu \neq 0$ *if and only if the coupling constants obey*

$$\alpha_3 = -2(\alpha_1 + \alpha_2), \qquad \alpha_5 = -\alpha_4. \tag{14}$$

κ *and* $\beta_1,\ \beta_2,\ \beta_3$ *being arbitrary.*

Proof. In the case of zero torsion there are some simplifications. Now quantities with and without tilde sign coincide, and it will be dropped. Due to the definition (7) one can easily transform Greek indices into Latin ones inside the covariant derivatives. The one more simplification arises from the additional symmetry of the curvature tensor 10.

Using the Bianchi identity (6) equation for the Lorentz connection (9) is transformed,

$$\nabla^c \left[\left(\alpha_1 + \alpha_2 + \frac{1}{2}\alpha_3 \right) R_{abcd} + \frac{1}{4}(\alpha_4 + \alpha_5)(R_{ac}\eta_{bd} - R_{ad}\eta_{bc} - R_{bc}\eta_{ad} + R_{bd}\eta_{ac}) \right] = 0 \tag{15}$$

where $\nabla^c = e^{\mu c}\nabla_\mu$. Taking the trace of this equation with η^{bd} and using the contracted Bianchi identity $\nabla_\mu R = 2\nabla_\nu R_\mu{}^\nu$ one obtains

$$\frac{1}{2}\left(\alpha_1 + \alpha_2 + \frac{1}{2}\alpha_3 + \alpha_4 + \alpha_5 \right) \nabla_a R = 0. \tag{16}$$

Using the Bianchi identity (5) in the form

$$\nabla^c C_{abcd} = \frac{1}{2}(\nabla_a R_{bd} - \nabla_b R_{ad}) - \frac{1}{12}(\nabla_a R \eta_{bd} - \nabla_b R \eta_{ad})$$

the traceless part of (15) can be rewritten as

$$2\left(\alpha_1 + \alpha_2 + \frac{1}{2}\alpha_3 + \frac{1}{4}\alpha_4 + \frac{1}{4}\alpha_5 \right) \nabla^c C_{abcd} = 0. \tag{17}$$

For $\nabla_a R \neq 0$ and $\nabla^c C_{abcd} \neq 0$ the relations between coupling constants (14) are the unique solution of (16) and (17). □

Note that due to the Bianchi identity the Ricci tensor under the conditions of Theorem 2 cannot be covariantly constant $\nabla_\nu R_\mu{}^\nu \neq 0$. Thus all irreducible components of the curvature are nonconstant.

Like in the previous case equation of motion for tetrad field under the restrictions (11) and (14) and zero torsion condition reduces to the Einstein equation (13). But now tetrad field will define nontrivial curvature, torsion of the manifold being zero. This is the Einsteinian limit of the model.

As was already mentioned the conditions of the proved theorems have direct physical interpretation in the physics of solids with continuously distributed defects. Theorem 1 yields the necessary condition for the existence of a space-time with only dislocations, while Theorem 2 yields the necessary conditions for the existence of a space-time with only disclinations. In fact, these conditions are sufficient because in both cases equation of motion for the tetrad field (8) coincides with the Einstein equation (13) which is known to have nontrivial solutions.

So the two conditions on the space of solution exclude five of the ten free parameters, and the resulting Lagrangian has the form

$$\frac{1}{e}L = \kappa \widetilde{R}(e) + c_1 C^A_{abcd} C^{Aabcd} + c_2 R^A_{ab} R^{Aab} + c_3 R^{*2} + \lambda \tag{18}$$

where $c_1 = -\frac{1}{4}(\alpha_1 - \alpha_2)$, $c_2 = -\frac{1}{2}(\alpha_1 - \alpha_2 + \alpha_4)$, $c_3 = 18(\alpha_1 + \alpha_2)$.

4 Conclusion

So we have proved that two simple and physically reasonable constraints on the space of solutions of dynamical torsion theory define five parameters Lagrangian. The resulting theory contains the Hilbert–Einstein Lagrangian, three specific square curvature terms (which equal zero in the case of zero torsion) and cosmological constant. The space of solutions of the equations of motion contains at least two sectors. The first is the celebrated Einstein's general relativity and the second is the less known Weitzenböck's absolute parallelism theory. Both sectors are described by the same dynamical variable that is the tetrad field, the Lorentz connection being excluded from the theory. In the Einsteinian limit the Lorentz connection is the known function of the tetrad, while in absolute parallelism theory Lorentz connection is a "pure gauge" and can be set equal to zero. In both cases the dynamics is governed by the Einstein equation for the tetrad which defines curvature in the Einsteinian limit and torsion in the absolute parallelism theory.

The Lagrangian 18 is highly degenerate. Besides the usual graviton it describes the massless Lorentz connection modes [6]. From Hamiltonian point of view it contains primary if-constraints that may define the generators of extra gauge symmetry [24]. The nonpropagating modes of the theory break 2 of the 10 Cauchy–Kowalevski conditions which are sufficient conditions for propagating of all modes in the theory [13]. The hyperbolicity conditions [13, 14] (also obtained for all modes) in our case lead to further restrictions $\kappa = c_1 = c_3 = 0$, c_2 being arbitrary. Because of the large number of nonpropagating modes and the possible existence of extra gauge symmetry the Lagrangian 18 needs a separate consideration. Nevertheless, by costruction the space of solutions of the equations of motion surely contains at least two well defined sectors both goverened by the Einstein equation which is hyperbolic and possesses well posed Cauchy problem [25].

Linear approximation for Lagrangian (18) was elucidated in [6]. It is interesting that for $c_2 = -2c_1$ and $\lambda = 0$ one obtains the unique Lagrangian without ghosts and tachyons found by Kuhfuss and Nitsch [6]. Another attractive feature is that the Lagrangian 18 is one of the particular Lagrangians yielding the asymptotically Newtonian behavior of the gravitational field found in Ref.[12]. The supersymmetric extension of this Lagrangian is discussed in [26].

The constraints discussed in the present paper were also considered in lower number of dimensions. In three dimensional case, which is relevant to a static media with dislocations and disclinations, one obtains the sensible two-parameter Lagrangian [20] which has physically reasonable solutions. Two-dimensional gravity with torsion [27, 28], which recently was proved to be completely integrable [22–24] , and renormalizable [32] admits solutions with zero torsion (only disclinations) but not admit solutions with constant curvature (only dislocations). Perhaps, the latter fact is connected with the absence of everywhere smooth vector field on a sphere, singularity of the vector field (director) representing disclination.

The author would like to thank I. V. Volovich for careful reading the manuscript and enlightening discussions.

References

[1] F. W. Hehl. Four lectures on Poincaré gauge field theory. In P. G. Bergman and V. de Sabbata, editors, *Spin, Torsion, Rotation and Supergravity*, New York, 1980. Plenum Press.

[2] D. E. Neville. Experimental bounds on the coupling of massless spin-1 torsion. *Phys. Rev.*, D25(2):573–576, 1982.

[3] W. R. Stoeger. The physics of detecting torsion and placing limits on its effects. *Gen. Rel. Grav.*, 17(10):981–988, 1985.

[4] E. Sezgin and P. van Nieuwenhuizen. New ghost-free gravity lagrangians with propagating torsion. *Phys. Rev.*, D21(12):3269–3280, 1980.

[5] E. Sezgin. Class of ghost-free gravity lagrangians with propagating torsion. *Phys. Rev.*, D24(6):1677–1680, 1981.

[6] R. Kuhfuss and J. Nitsch. Propagating modes in gauge field theories of gravity. *Gen. Rel. Grav.*, 18(12):1207–1227, 1986.

[7] K. Hayashi and T. Shirafuji. Gravity from Poincaré gauge theory of the fundamental particles. IV. Mass and energy of particle spectrum. *Prog. Theor. Phys.*, 64(6):2222–2241, 1980.

[8] S. Miyamoto, T. Nakano, T. Ohtani, and Y. Tamura. Linear approximation for the lorentz gauge field. *Prog. Theor. Phys.*, 66(2):481–497, 1981.

[9] S. Ramaswamy and P. B. Yasskin. Birkhoff theorem for an $R + R^2$ theory of gravity with torsion. *Phys. Rev.*, D19(8):2264–2267, 1979.

[10] D. E. Neville. Birkhoff theorems for $R+R^2$ gravity theories with torsion. *Phys. Rev.*, D21(10):2770–2775, 1980.

[11] R. T. Rauch, J. C. Shaw, and H.-T. Nieh. Birkhoff's theorem for ghost-free, tachyon-free $R+R^2+Q^2$ theories with torsion. *Gen. Rel. Grav.*, 14(4):331–354, 1982.

[12] H.-H. Chen, D.-C. Chern, R.-R. Hsu, J. M. Nester, and W. B. Yeund. Asymptotically Newtonian conditions for Poincaré gauge theory. *Prog. Theor. Phys.*, 79(1):77–85, 1988.

[13] A. Dimakis. The initial value problem of the Poincaré gauge theory in vacuum I. Second order formalism. *Ann. Inst. Henri Poincaré*, 51A(4):371–388, 1989.

[14] A. Dimakis. The initial value problem of the Poincaré gauge theory in vacuum II. First order formalism. *Ann. Inst. Henri Poincaré*, 51A(4):389–417, 1989.

[15] K. Hayashi and T. Shirafuji. New general relativity. *Phys. Rev.*, D19(12):3524–3553, 1979.

[16] F. Müller-Hoissen and J. Nitsch. On the tetrad theory of gravity. *Gen. Rel. Grav.*, 17(8):747–760, 1985.

[17] E. Kröner. Continuum theory of defects. In R. Balian et al., editor, *Less Houches, Session XXXV, 1980 – Physics of Defects*, pages 282–315. North-Holland Publishing Company, 1981.

[18] A. Kadić and D. G. B. Edelen. *A gauge theory of dislocations and disclinations.* Springer–Verlag, Berlin – Heidelberg, 1983.

[19] J. D. McCrea, F. W. Hehl, and E. W. Mielke. Mapping Noether identities into Bianchi identities in general relativistic theories of gravity and in the field theory of static lattice defects. *Int. J. Theor. Phys.*, 29(11):1185–1206, 1990.

[20] M. O. Katanaev and I. V. Volovich. Theory of defects in solids and three-dimensional gravity. *Ann. Phys.*, 216(1):1–28, 1992.

[21] S. Kobayashi and K. Nomizu. *Foundations of differential geometry*, volume 1. Interscience publishers, New York – London, 1963.

[22] F. W. Hehl, P. von der Heyde, G. D. Kerlick, and J. M. Nester. General relativity with spin and torsion: foundations and prospects. *Rev. Mod. Phys.*, 48(3):393–416, 1976.

[23] S. M. Christensen. Second- and fourth-order invariants on curved manifolds with torsion. *J. Phys.*, A13(9):3001–3009, 1980.

[24] M. Blagojević and M. Vasilić. Extra gauge symmetries in a weak-field approximation of an $R + T^2 + R^2$ theory of gravity. *Phys. Rev.*, D35(12):3748–3759, 1987.

[25] Y. Choquet-Bruhat. In W. Rindler and A. Trautman, editors, *Festschrift for I. Robinson*, Napoli, 1987. Bibliopolis.

[26] C. Castro. A supersymmetric Lagrangian for Poincaré gauge theories of gravity. *Prog. Theor. Phys.*, 82(3):616–630, 1989.

[27] M. O. Katanaev and I. V. Volovich. String model with dynamical geometry and torsion. *Phys. Lett.*, 175B(4):413–416, 1986.

[28] M. O. Katanaev and I. V. Volovich. Two-dimensional gravity with dynamical torsion and strings. *Ann. Phys.*, 197(1):1–32, 1990.

[29] M. O. Katanaev. Complete integrability of two-dimensional gravity with dynamical torsion. *J. Math. Phys.*, 31(4):882–891, 1990.

[30] M. O. Katanaev. Conformal invariance, extremals, and geodesics in two-dimensional gravity with torsion. *J. Math. Phys.*, 32(9):2483–2496, 1991.

[31] W. Kummer and D. J. Schwarz. General analytic solution of R^2–gravity with dynamical torsion in two dimensions. Preprint, Technische Universität Wien, TUW-91-08, to appear in *Phys. Rev. D.*

[32] W. Kummer and D. J. Schwarz. Renormalization of R^2–gravity with dynamical torsion in $d = 2$. Preprint, Technische Universität Wien, TUW-91-09.